

Mathematics

Mr Pythagoras



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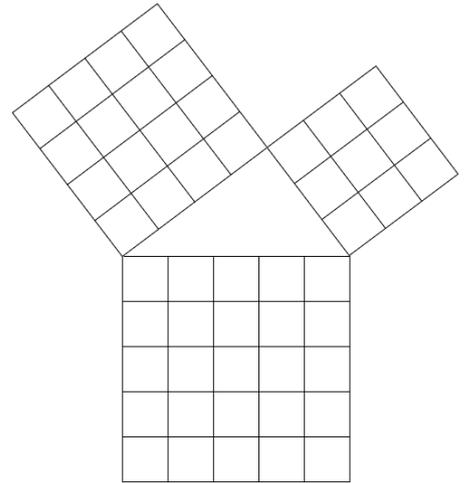
Pythagoras

You might know that Pythagoras is the famous Greek mathematician who told everyone that if you make squares on each side of a right-angled triangle then the area of the square on the triangle's hypotenuse (the longest side) will always equal the sum of the two squares on the triangle's other two sides.

Look at the diagram on the right.

The triangle's hypotenuse is 5 units in length and the other two sides measure 3 and 4 units.

The square drawn on the hypotenuse is 25 square units (5x5) in area; this equals the sum of the other two sides' areas... 16 (4x4) plus 9 (3x3).



The triangle above (sides 3, 4 and 5) is just one of an infinite number of right-angled triangles. This triangle (3-4-5) is known as a Pythagorean triplet.

Another right-angled triangle is the one with sides 5, 12 and 13 (because $13^2 = 12^2 + 5^2$); 5-12-13 is another Pythagorean triplet.

Two other Pythagorean triplets are 7-24-25 (because $25^2 = 24^2 + 7^2$) and 9-40-41 (because $41^2 = 40^2 + 9^2$).

The first six Pythagorean triplets (NB: there are other side-length combinations that are found in right-angled triangles but Pythagorean triplets are the only ones with three whole numbers).

<i>side 1</i>	<i>side 2</i>	<i>side 3</i>
3	4	5
5	12	13
7	24	25
9	40	41
11	60	61
13	84	85

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Class or Group Activity: Let's say your friend tells you they know of a Pythagorean triplet. They then say they will tell you the length of just one of the sides but you have to work out the lengths of the other two sides. Can you do it? Yes, it's easy! Read on.

In the table above, can you see any patterns? (there are several). Notice that the middle side is always an even number and that the shortest and longest sides are always odd. Note too that the longest side (the hypotenuse) is always just one unit more than the middle side).

Now, back to your friend, the one who gave you the length of just one side of a right-angles triangle and asked you to come up with the other two sides. By noticing patterns in the Pythagorean triplets table you might just have figured out a way to do it.

Solutions:

Say your friend gives you a side with an even number and asks you to complete the triplet with the other two sides. Simply add 1 to the even number and that will give you the hypotenuse. To work out the 3rd side (in this case, the smallest side) all you have to do is get the square root of the sum of the two sides you already know.

Example 1:

Your friend tells you that the side he/she is thinking of is 12 units in length.
12 is an even number so add 1 to 12 and you get 13.
Now you have two sides of the triplet...12 and 13.
Add 12 and 13 together and you get 25.
Now get the square root of 25, which is 5.
Your Pythagorean triplet is now complete and the sides are 5, 12 and 13.

Example 2:

Your friend tells you that the side he/she is thinking of is 11 units in length.
11 is midway between 10 and 12 so multiply 10 and 12 together to get 120.
Half of 120 is 60 which is the 2nd number of the triplet.
The square root of $13^2 + 60^2$ is 61.
You have now completed the triplet... 11, 60, 61.

Example 3:

Your friend tells you that the side he/she is thinking of is 41 units in length.
41 is midway between 40 and 42 so multiply 40 and 42 together to get 1 680.
Half of 1 680 is 840 which is the 2nd number of the triplet.
As 840 is an even number the other side must be 840+1 units in length.
Your Pythagorean triplet is now complete and the sides are 41, 840 and 841.

